Easter revision

Paper 1 Section A

Each correct answer in this section is worth two marks.

1. What is the distance, in units, between the points $(-1, 2)$ and $(4, 5)$?

   A. $\sqrt{8}$

   B. $\sqrt{16}$

   C. $\sqrt{34}$

   D. $\sqrt{58}$

2. What is the distance, in units, between the points (a, b) and $(-b, a)$?

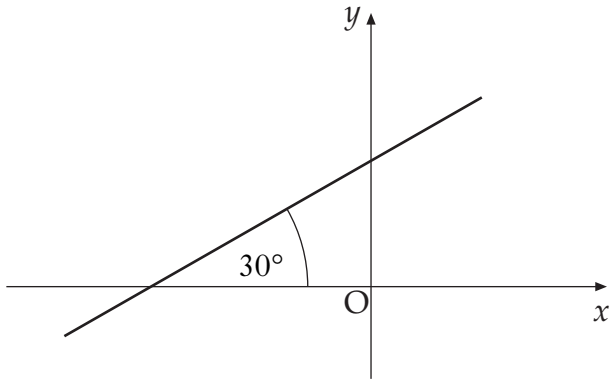
   A. $\sqrt{2}\sqrt{a^2 + b^2}$

   B. $\sqrt{2}(a + b)$

   C. $\sqrt{2}\left(\sqrt{a} + \sqrt{b}\right)$

   D. $2\sqrt{a^2 + b^2}$

3. A line makes an angle of $30°$ with the positive direction of the x-axis as shown.



   What is the gradient of the line?

   A. $\frac{1}{\sqrt{3}}$

   B. $\frac{1}{\sqrt{2}}$

   C. $\frac{1}{2}$

   D. $\frac{\sqrt{3}}{2}$

4. The line through the points $(-2, 5)$ and $(7, a)$ has gradient 3.

   What is the value of a?

   A. 8

   B. 22

   C. 28

   D. 32

5. A line L is perpendicular to the line with equation $2x - 3y - 6 = 0$.

   What is the gradient of the line L?

   A. $-\frac{3}{2}$

   B. $-\frac{1}{2}$

   C. $\frac{2}{3}$

   D. 2

6. The line with equation $y = ax + 4$ is perpendicular to the line with equation $3x + y + 1 = 0$.

   What is the value of a?

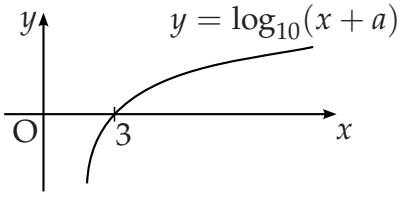
   A. -3

   B. $-\frac{1}{3}$

   C. $\frac{1}{3}$

   D. 3

7. The diagram shows the graph of $y = \log_{10}(x + a)$.



   What is the value of a?

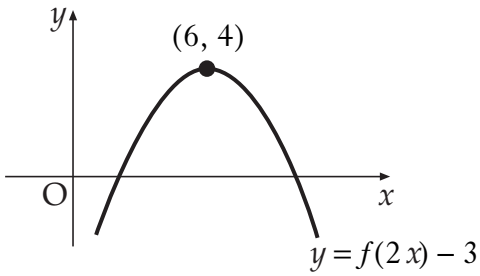
   A. -3

   B. -2

   C. 2

   D. 3

8. The diagram shows the graph of $y = f(2x) - 3$.



   What are the coordinates of the turning point on the graph of $y = f(x)$?

   A. $(12, 7)$

   B. $(12, 1)$

   C. $(3, 7)$

   D. $(3, 1)$

9. Two functions f and g are defined by $f(x) = 4x + 1$ and $g(x) = x^2 - 2$.

   Find a formula for $f(g(x))$.

   A. $4x^2 - 7$

   B. $4x^2 - 1$

   C. $16x^2 + 8x - 1$

   D. $4x^3 + x^2 - 8x - 2$

10. If $x^2 - 8x + 7$ is written in the form $(x - p)^2 + q$, what is the value of q?

    A. -9

    B. -1

    C. 7

    D. 23

11. A function f is defined by $f(x) = x^3 + kx^2 + 2x$.

    Given that $f'(2) = 26$, what is the value of k?

    A. 3

    B. $\frac{7}{2}$

    C. 5

    D. 10

12. What is the gradient of the tangent to the curve $y = 4x^3 + x^2 + 3$ at $x = 2$?

    A. $24\frac{2}{3}$

    B. 39

    C. 52

    D. 55

13. A sequence is defined by the recurrence relation $u_{n+1} = au_n + b$, where a and b are constants.

    Given that $u_0 = 4$ and $u_1 = 8$, find an expression for a in terms of b.

    A. $a = \frac{1}{2} - \frac{1}{8}b$

    B. $a = 2 - \frac{1}{4}b$

    C. $a = \frac{1}{2} + \frac{1}{8}b$

    D. $a = 2 + \frac{1}{4}b$

14. A vector v is given by $\begin{pmatrix} -3 \\ 2 \\ 6 \end{pmatrix}$.

    What is the length, in units, of $3v$?

    A. 7

    B. 15

    C. 21

    D. 49

15. The vectors $\begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix}$ and $\begin{pmatrix} k \\ 2 \\ -1 \end{pmatrix}$ are perpendicular.

    What is the value of k?

    A. -3

    B. 3

    C. $\frac{10}{3}$

    D. $\frac{8}{3}$

16. Differentiate $2(4 - x)^{-\frac{1}{2}}$ with respect to x.

    A. $(4 - x)^{-1}$

    B. $-(4 - x)^{-1}$

    C. $(4 - x)^{-\frac{3}{2}}$

    D. $-(4 - x)^{-\frac{3}{2}}$

17. Find $\int (2x^{-4} + \cos 5x)\, dx$.

    A. $-\frac{2}{5}x^{-5} - 5\sin 5x + c$

    B. $-\frac{2}{5}x^{-5} + \frac{1}{5}\sin 5x + c$

    C. $-\frac{2}{3}x^{-3} + \frac{1}{5}\sin 5x + c$

    D. $-\frac{2}{3}x^{-3} - 5\sin 5x + c$

18. Given that $f(x) = 3\cos(2x)$, what is the value of $f'\left(\frac{\pi}{6}\right)$?

A. 3

B. $-3\sqrt{3}$

C. -3

D. $\frac{3\sqrt{3}}{2}$

19. Simplify $\log_4 8 + \log_4 2 - 3\log_5 5$.

A. $-\frac{1}{2}$

B. -1

C. $\log_4\left(\frac{16}{5}\right)$

D. $\log_4\left(\frac{16}{125}\right)$

20. Solve $\log_a 5 + \log_a x = \log_a 20$ for $x > 0$.

A. $x = \frac{1}{4}$

B. $x = 4$

C. $x = 15$

D. $x = 100$

21. Solve $3\log_a 2 = \frac{1}{2}$ for a.

A. $a = 64$

B. $a = 36$

C. $a = \frac{4}{9}$

D. $a = \frac{1}{16}$

22. The point $(2, -3)$ lies on the circle with equation $x^2 + y^2 + 6x - 2y + c = 0$.

What is the value of c?

A. -31

B. -13

C. -1

D. 9

23. A circle has centre $(2, 4)$ and passes through $(-1, 1)$.

What is the equation of the circle?

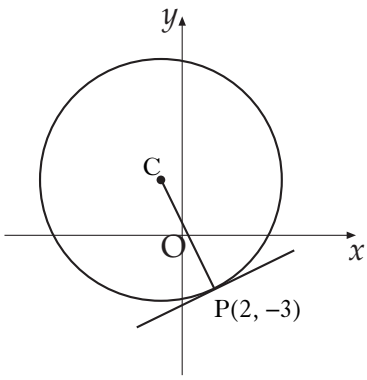
A. $(x-2)^2 + (y-4)^2 = \sqrt{18}$

B. $(x-2)^2 + (y-4)^2 = 18$

C. $(x+2)^2 + (y+4)^2 = 18$

D. $(x+2)^2 + (y+4)^2 = 26$

24. The point P$(2, -3)$ lies on the circle with centre C as shown.



The gradient of CP is -2.

What is the equation of the tangent at P?

A. $y + 3 = -2(x - 2)$

B. $y - 3 = -2(x + 2)$

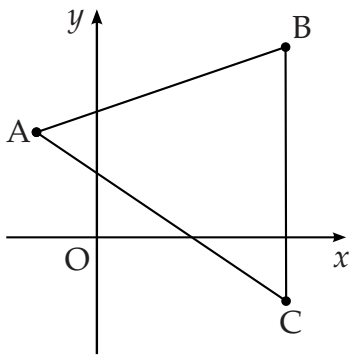
C. $y + 3 = \frac{1}{2}(x - 2)$

D. $y - 3 = \frac{1}{2}(x + 2)$

[END OF PAPER 1 SECTION A]

Paper 1 Section B

25. Triangle ABC has vertices $A(-3,5)$, $B(9,9)$ and $C(9,-3)$.



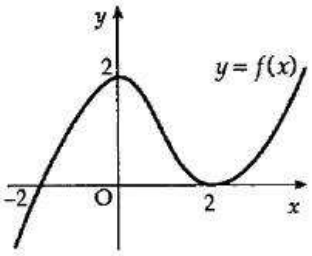
(*a*) Write down the equation of BC. **1**

(*b*) Find the equation of the altitude from A. **2**

(*c*) Find the equation of the perpendicular bisector of AB. **4**

(*d*) Find where the perpendicular bisector of AB and the altitude from A intersect. **2**

[SQA] 26. The diagram shows the graph of $y = f(x)$, where $-2 \le x \le 3$.
On separate diagrams, sketch the graphs of



(*a*) $y = -f(x)$; **2**

(*b*) $y = f'(x)$. **3**

[SQA] 27. $f(x) = 2x - 1$, $g(x) = 3 - 2x$ and $h(x) = \frac{1}{4}(5 - x)$.

(*a*) Find a formula for $k(x)$ where $k(x) = f\big(g(x)\big)$. **2**

(*b*) Find a formula for $h\big(k(x)\big)$. **2**

(*c*) What is the connection between the functions h and k? **1**

[SQA] 28. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. **2**

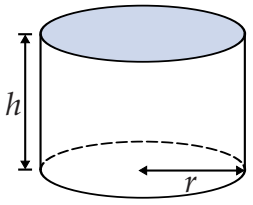
(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

29. A cylindrical water tank, with solid top and bottom, has radius r metres and height h metres.
The surface area of the tank is 4 square metres.



(*a*) (i) Find an expression for h in terms of r.

(ii) Hence show that the volume, V cubic metres, of the tank is given by

$$V = r(2 - \pi r^2).$$ **4**

(*b*) Find the exact value of r for which the volume V is a maximum. **5**

[SQA] 30. If $y = x^2 - x$, show that $\dfrac{dy}{dx} = 1 + \dfrac{2y}{x}$. **3**

[SQA] 31. Find $\dfrac{dy}{dx}$ where $y = \dfrac{4}{x^2} + x\sqrt{x}$. **4**

[SQA] 32. Find the equation of the tangent to the curve $y = 3x^2 + 2$ at the point where $x = 1$. **4**

[SQA] 33.

(*a*) The function f is defined by $f(x) = x^3 - 2x^2 - 5x + 6$.
The function g is defined by $g(x) = x - 1$.
Show that $f\big(g(x)\big) = x^3 - 5x^2 + 2x + 8$. **4**

(*b*) Factorise fully $f\big(g(x)\big)$. **3**

(*c*) The function k is such that $k(x) = \dfrac{1}{f\big(g(x)\big)}$.
For what values of x is the function k not defined? **3**

34. The circles centred at A and B have equations $x^2 + y^2 + 8x + 12y + 36 = 0$ and $x^2 + y^2 - 4x - 4y - 28 = 0$ respectively.

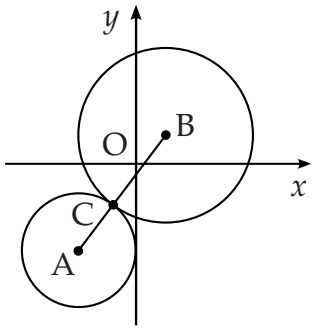
(*a*) Write down the coordinates of A and B. **2**

(*b*) Show that the circles touch externally. **4**

(*c*) The circles touch at point C.

(i) Find the ratio in which C divides AB.

(ii) Hence find the coordinates of C. **4**



[SQA] 35. Differentiate $\sin 2x + \dfrac{2}{\sqrt{x}}$ with respect to x. **4**

36. The function f, defined on a suitable domain, is such that $f'(x) = \dfrac{1}{\sqrt{(1+x)^3}}$.

Given that $f(3) = -1$, express $f(x)$ in terms of x. **5**

[SQA] 37. Given $x = \log_5 3 + \log_5 4$, find algebraically the value of x. **4**

[SQA] 38. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

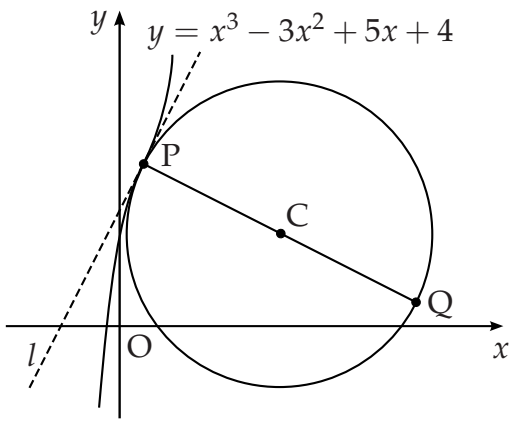
(*a*) (i) Show that the radius of circle P is $4\sqrt{2}$.

(ii) Hence show that circles P and Q touch. **4**

(*b*) Find the equation of the tangent to the circle Q at the point $(-4, 1)$. **3**

(*c*) The tangent in (*b*) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing you answers in the form $a \pm b\sqrt{3}$. **3**

39. The diagram below shows the graph of the cubic with equation $y = x^3 - 3x^2 + 5x + 4$ and a circle with centre C.



At the point P the line l is a tangent to both the curve and the circle.

(*a*) The tangent line l has gradient 2. Find the coordinates of P. **5**

(*b*) The circle has equation $x^2 + y^2 - 14x - 8y + c = 0$.

Determine the value of c. **2**

(*c*) The line PQ is a diameter of the circle. Determine the coordinates of Q. **2**

[END OF PAPER 1 SECTION B]

Paper 2

1. The function f is defined by $f(x) = x^3 + px^2 + qx + 3$.

   The tangent to the curve $y = f(x)$ at $x = 1$ has gradient -3.

   (*a*) Show that $2p + q = -6$. **3**

   (*b*) Given that 3 is a root of the equation $f(x) = 0$, find the values of p and q. **4**

[SQA] 2. A sequence is defined by the recurrence relation $u_n = 0{\cdot}9u_{n-1} + 2$, $u_1 = 3$.

   (*a*) Calculate the value of u_2. **1**

   (*b*) What is the smallest value of n for which $u_n > 10$? **1**

   (*c*) Find the limit of this sequence as $n \to \infty$. **2**

[SQA] 3. **(*a*)** **At 12 noon a hospital patient is given a pill containing 50 units of antibiotic.**
By 1 pm the number of units in the patient's body has dropped by 12%.
By 2 pm a further 12% of the units remaining in the body at 1 pm is lost.
If this fall-off rate is maintained, find the number of units of antibiotic remaining at 6 pm. **(4)**

(*b*) **A doctor considers prescribing a course of treatment which involves a patient taking one of these pills every 6 hours over a long period of time.**
The doctor knows that mopre than 100 units of this antibiotic in the body is regarded as too dangerous.
Should the doctor prescribe this course of treatment?
Give reasons for your answer. **(6)**

[SQA] 4. ABCD is a quadrilateral with vertices A$(4, -1, 3)$, B$(8, 3, -1)$, C$(0, 4, 4)$ and D$(-4, 0, 8)$.

   (*a*) Find the coordinates of M, the midpoint of AB. **1**

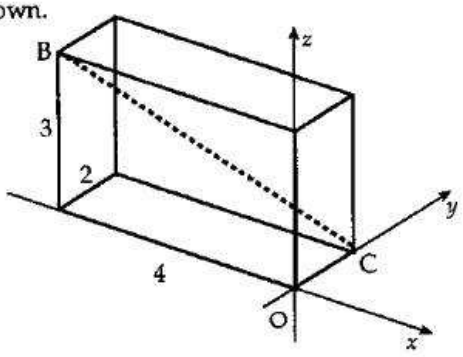
   (*b*) Find the coordinates of the point T, which divides CM in the ratio 2 : 1. **3**

   (*c*) Show that B, T and D are collinear and find the ratio in which T divides BD. **4**

[SQA] 5. A cuboid crystal is placed relative to the coordinate axes as shown.

(*a*) Write down $\overrightarrow{BC}$ in component form.

(*b*) Calculate $\left|\overrightarrow{BC}\right|$. 2

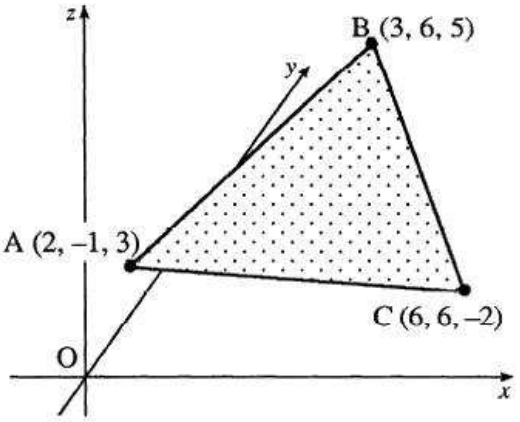


[SQA] 6. A triangle ABC has vertices
A (2, –1, 3), B(3, 6, 5) and C (6, 6, –2).

(*a*) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Calculate the size of angle BAC. **(5)**

(*c*) Hence find the area of the triangle. **(2)**



[SQA] 7. VABCD is a pyramid with rectangular base ABCD.

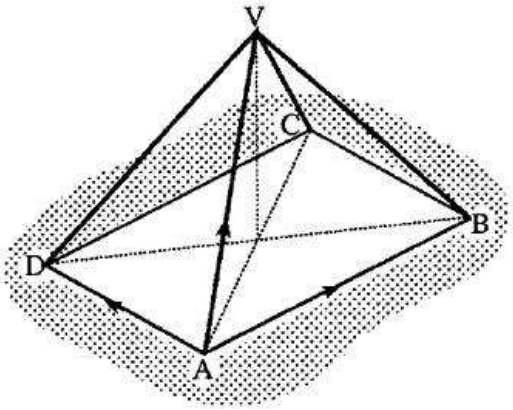
The vectors $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AV}$ are given by

$\overrightarrow{AB} = 8i + 2j + 2k$

$\overrightarrow{AD} = -2i + 10j - 2k$ and

$\overrightarrow{AV} = i + 7j + 7k$.

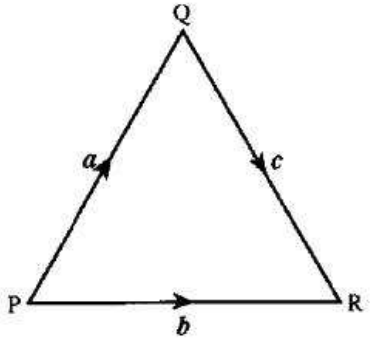
Express $\overrightarrow{CV}$ in component form. 3



[SQA] 8. PQR is an equilateral triangle of side 2 units.

$\overrightarrow{PQ} = \boldsymbol{a}$, $\overrightarrow{PR} = \boldsymbol{b}$ and $\overrightarrow{QR} = \boldsymbol{c}$.

Evaluate $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$ and hence identify two vectors which are perpendicular. 4



[SQA] 9. (*a*) A tractor tyre is inflated to a pressure of 50 units.
Twenty-four hours later the pressure has dropped to 10 units.

If the pressure, P_t units, after t hours is given by the formula $P_t = P_0 e^{-kt}$, find the value of k, to three decimal places. **(5)**

(*b*) The tyre manufacturer advises that serious damage to the tyre will result if it is used when the pressure drops below 30 units.

If the farmer inflates the tyre to 50 units and drives the tractor for four hours, can the tractor be driven further without inflating the tyre and without risking serious damage to the tyre? **(4)**

[SQA] 10. (*a*) Express $\sin x° - 3\cos x°$ in the form $k\sin(x-a)°$ where $k > 0$ and $0 \le a < 360$. Find the values of k and a. **4**

(*b*) Find the maximum value of $5 + \sin x° - 3\cos x°$ and state a value of x for which this maximum occurs. **2**

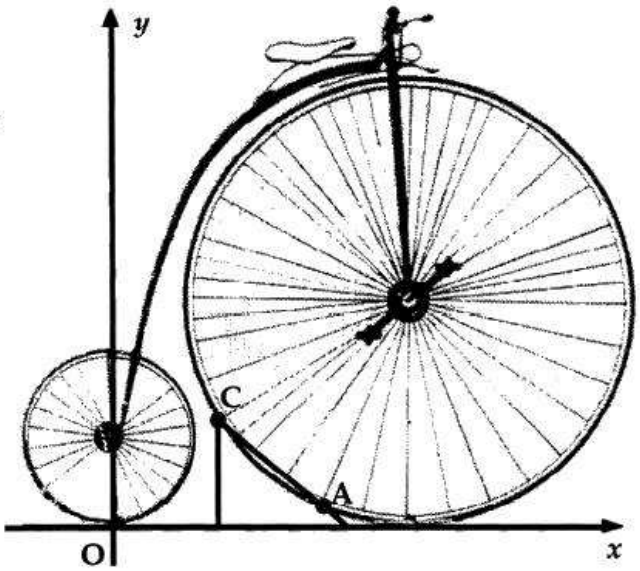
[SQA] 11.

(*a*) Show that $2\cos(x° + 30°) - \sin x°$ can be written as $\sqrt{3}\cos x° - 2\sin x°$. **3**

(*b*) Express $\sqrt{3}\cos x° - 2\sin x°$ in the form $k\cos(x° + \alpha°)$ where $k > 0$ and $0 \le \alpha \le 360$ and find the values of k and α. **4**

(*c*) Hence, or otherwise, solve the equation $2\cos(x° + 30°) = \sin x° + 1$, $0 \le x \le 360$. **3**

[SQA] 12. A penny-farthing bicycle on display in a museum is supported by a stand at points A and C. A and C lie on the front wheel.
With coordinate axes as shown and 1 unit = 5cm, the equation of the rear wheel (the small wheel) is $x^2 + y^2 - 6y = 0$ and
the equation of the front wheel is $x^2 + y^2 - 28x - 20y + 196 = 0$.



(*a*) (i) Find the distance between the centres of the two wheels.
(ii) Hence calculate the clearance, i.e. the smallest gap, between the front and rear wheels. Give your answer to the nearest millimetre. **(8)**

(*b*) B(7,3) is half-way between A and C, and P is the centre of the front wheel.
(i) Find the gradient of PB.
(ii) Hence find the equation of AC and the coordinates of A and C. **(8)**

[SQA] 13. Solve the equation $3\cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \leq x \leq 360$. **5**

[END OF PAPER 2]